



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

REPORT DOCUMENTATION PAGE		BEFORE COMPLETING FORM
	2. GOVT ACCESSION NO	3 RECIPIENT'S CATALOG NUMBER
FOSR-TR- 87-0211		
4. TITLE (and Subtitle)		5 TYPE OF REPORT & PERIOD COVERED
Control and Identification of Time Varying Systems		Final Technical Report
		July 1, 1985-Aug. 31, 1986
		6 PERFORMING ORG REPORT NUMBER
7. AUTHOR(s)		8 CONTRACT OR GRANT NUMBER(#)
Allan E. Pearson		
Principal Investigator		AFOSR-85-0300
9. PERFORMING ORGANIZATION NAME AND ADDRESS		10. PROGRAM ELEMENT PROJECT TASK AREA & WORK UNIT NUMBERS
Professor A. E. Pearson		222445
Division of Engineering, Brown University		2304/AL
Providence, RI 02912		12. REPORT DATE
Air Force Office of Scientific Research/NM		October 3, 1986
Building 410, Bolling AFB, DC 20332		13. NUMBER OF PAGES
		4
14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)		15 SECURITY CLASS. (of this report)
		Unclassified
		15- DECLASSIFICATION DOWNGRADING SCHEDULE
16 DISTRIBUTION STATEMENT (of this Report)		
DRIVING TO SERVICE OF THE PROPERTY OF THE PROP		
DISTRIBUTION ST	ATEMENT A	
Approved for pub	slia releases	
Distribution U	blimited	
12 00700	n Black 20 of defense of	Renaul .
17 DISTRIBUTION STATEMENT (of the abstract entered)	a proce 20, ii different froi	DIC
		LECTE
		MAR 0 3 1987
18 SUPPLEMENTARY NOTES		
		<b>1 D</b>
		•
19 KEY WORDS (Continue on reverse side if necessary and	d identify by block numbers	
Distributed and point-delay time-lag control	systems. Feedback	stabilization, state reconstruction, and
tracking controllers for time-lag systems. P	Parameter identification	on of linear, bilinear and polynomia

ت س

O ABSTRACT continue on reverse side if necessary and identify by block number

Research is summarized for the state feedback stabilization of point delay and distributed delay time lag control systems via the development of a reducing transformation technique. The approach facilitates the controller design using well established delay free methods once the unstable pole set is delineated for the time-lag system. Research is described on the dual problem of state reconstruction using input output data and also on a tracking problem which employs integral action for the controller. Research is described in the use of a Fourier based modulating function technique for the least squares parameter identification of a class of polynomial input output differential systems.

DD . JAN 73 1473 EDITION OF I NOV 65 IS OBSOLETE

++

input-output differential systems

F. ALTY . ASSIER ATION OF THIS PAGE When Date Entered

# AFOSR-TM- 87-0211

Final Technical Report
to the
Directorate of Mathematical and Information Sciences
Department of the Air Force
Air Force Office of Scientific Research (AFSC)
AFOSR/NM, Building 410
Bolling Air Force Base
Washington, D.C. 20332

For the Grant AFOSR 85-0300

# CONTROL AND IDENTIFICATION OF TIME VARYING SYSTEMS

Period Covered: July 1, 1985 to August 31, 1986

from

Allan E. Pearson
Division of Engineering
Brown University
Providence, RI 02912

Report prepared by:

Allan E. Pearson

Professor of Engineering

allen & June

Principal Investigator

Carl Cometta

**Executive Officer** 

Division of Engineering

September 1986

#### 1. Introduction

This final technical report covers a one year period preceding August 31, 1986 during which support was provided under AFOSR Grant 85-0300. The research results described in Section 3 below were partially described in the AFOSR Proposal No. 86-NM-191 which is pending.

### 2. List of Scientific Collaborators

Y. A. Fiagbedzi\*

F. C. Lee

A. E. Pearson\*

Post-doctoral Research Associate Former Graduate Student

Professor and Principal Investigator

## 3. Completed Research

The published research consists of three journal articles [1-3] and five conference proceedings papers [4-8]. There are three in-print or submitted-for-publication articles [9-11]. The results are described under the following headings.

## 3.1. Feedback Control and State Reconstruction of Time Lag Systems [3,6-11]

State feedback stabilization for the class of time lag systems described by the n-dimensional "point delayed" state and output equations

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-r) + B_0 \mu(t) + B_1 \mu(t-h_1)$$
 (1)

$$y(t) = C_0 x(t) + C_1 x(t - h_2)$$
 (2)

has been developed in [3,6] via a "reducing transformation" method. This technique has its roots in the classical Smith regulator for stabilizing systems characterized by transfer functions with pure transport lag, but until the work in [3,6] the approach had been restricted to differential delay systems with delays in the control variables only. The distinguishing feature of this technique is that once the reducing transformation has been found, the design of the stabilizing feedback control can be carried out using well known delay-free methods. At the heart of finding the reducing transformation for a given system (1) is the "characteristic matrix equation" which is defined as the  $n \times n$  matrix equation:

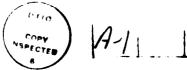
$$A - A_0 - e^{-rA} A_1 = 0. (3)$$

Based on a partitioning of the unstable and poorly damped pole set for (1) into N disjoint sets of n eigenvalues each, it is shown in [3] how to construct N real matrix solutions to (3), each of which inherits n eigenvalues from the unstable pole set, in terms of the corresponding N sets of  $n \times n$  modal matrices. Each modal matrix is comprised of n linearly independent left eigenvectors corresponding to the n eigenvalues in each set. In this sense, (3) is referred to as the "left" characteristic matrix equation for (1). This terminology is suggested by our recent development of a corresponding observer theory for (1) using the output measurement equation (2) [11]. Here the pertinent matrix equation is

$$F - A_0 - A_1 e^{-rF} = 0. (4)$$

The algorithm devised for solving (4) is the same as the one for solving (3) only involving *right* eigenvectors instead of left; hence, the terminology in reference to (3) and (4) as the "left" and "right"

<sup>&</sup>lt;sup>1</sup> For example, the treatment in Kwon and Pearson: "Feedback Stabilization of Linear Systems with Delayed Control," *IEEE Trans. on Auto. Contr.*, AC-25, no. 2, pp. 266-269, 1980.



tes

<sup>\*</sup> Received partial support under AFOSR 85-0300.

characteristic matrix equations respectively. Together, the ability to construct solutions to these equations using well known eigenvector techniques constitutes the first step of the ensuing controller/observer design methodology outlined in [3,11] using well established delay-free methods.<sup>2</sup> We believe that these equations will prove fundamental to many other system theoretic problems involving delay equations such as system approximation and filtering theory.

Extensions have been made in [8,9] to more general systems such as the class of distributed delay systems described by

$$\dot{x}(t) = \int_{-\tau}^{0} d\alpha(\theta)x(t+\theta) + \int_{-h}^{0} d\beta(\tau)u(t+\tau)$$
 (5)

where  $\alpha(\cdot)$  and  $\beta(\cdot)$  are matrix valued functions of bounded variation and the integrals are of the Stieltjes type. This includes multiple point delayed systems akin to the system (1) as special cases. Some results on the state feedback stabilization problem for (5) are contained in [8,9], while others pertaining to the observer theory are still under development.

A tracking theory for the system (5) has been advanced in [7,10] which facilitates integral action in the state feedback controller so that designated outputs will track step command inputs with zero steady state error while accomplishing stabilization with a prescribed degree of stability. It is intended to extend the theory by including an observer. When this extension is complete, it will represent a modern state space approach to the design of controllers for time lag systems which utilizes well-known delay-free methods, much in the same spirit as the design methodology for the classical Smith Predictor Controller. However, the work will by no means be complete at that point since, for example, we shall want to investigate issues of sensitivity and disturbance rejection, as well as perform a comparison with other approaches to the same class of control system problems such as those based on linear semigroup theory, the polynomial ring ideas, etc.

## 3.2. System Parameter Identification [1,2,4,5]

A least squares parameter identification technique has been formulated in [1] for the class of non-linear systems modeled by the polynomial input-output differential equation:

$$p^{n}y(t) + \sum_{i=1}^{n} \sum_{j=0}^{m} \sum_{k=0}^{m} a_{i}(j,k)p^{n-i}[u(t)]^{j}[y(t)]^{k} = 0$$
(6)

$$0 \le t \le T$$
,  $a_i(0,0) = 0$ ,  $i=1..n$ .

Here p is the differential operator d/dt and the  $a_i(j,k)$  represent parameters which are to be determined for a presumed given order n based on the input-output data [u(t),y(t)] observed over a single time interval [0,T] for a one-shot estimate, or over a sequence of time intervals, each of duration T, for sequential least squares. The basis of the technique is Shinbrot's method of moment functionals using trigonometric modulating functions. It is shown in [1] how the least squares identification can be formulated in a way that utilizes the computationally efficient FFT algorithm at each stage while avoiding the necessity to estimate unknown initial conditions for time limited data. In addition to the order of the system model and the number of parameters to be identified, the choice in modulating functions can be based to some extent on noise considerations, though much more remains to be done in this regard.

<sup>&</sup>lt;sup>2</sup> Actually, the first step is to check the spectral controllability/observability properties of (1) and (2) relative to the unstable pole set. However, this can be carried out using finite dimensional eigenvalue-eigenvector tests (the so called PBH tests) once the unstable pole set has been delineated.

A special case of the model (6) is the bilinear input-output system

$$p^{n}y(t) + \sum_{i=1}^{n} \alpha_{i}p^{n-i}y(t) = \sum_{i=1}^{n} p^{n-i} [\beta_{i}u(t) + \gamma_{i}u(t)y(t)]$$
 (7)

where the  $(\alpha_i, \beta_i, \gamma_i)$  represent parameters to be identified. Within the framework of the modulating function approach and using sinusoidal probing signals on sequential time intervals, it was shown in [4] how the least squares identification of the system parameters can be accomplished using essentially the same underlying computation as would attend the identification of a linear differential system of the same order. The order determination problem for (7) was formulated in [5] with the order n to be determined in addition to the system parameters. However, in retrospect this initial formulation is considered to be a failure and a new formulation is planned for future investigation based on the singular value decomposition and "total" least squares theory.

Although the parameter identification problem for linear systems is ostensibly a special case of the problems discussed in [1,4], more substantive results are obtained in [2] for handling noise corrupted data. Specifically, it is shown how the frequency domain interpretation can be beneficial in enhancing the signal to noise ratio of the modulated data for the deterministic least squares estimate Further, a maximum likelihood estimate is developed for the stochastic case of additive white gaussian noise in the data which effectively removes the bias when the parameter identification is considered in a recursive mode.

Future research in this area is planned on the problems of structure determination, e.g., determining the order n of the differential operator models in (6) and (7), and an extension of the theory to handle the deleterious effect caused by sensor dynamics.

### 4. Publications Under AFOSR 85-0300

### 4.1. Journal Articles

- [1] Pearson, A. E. and F. C. Lee, "On the identification of polynomial input-output differential systems," *IEEE Trans. on Auto. Contr.*, AC-30, no. 8, pp. 778-782, August, 1985.
- [2] Pearson, A. E. and F. C. Lee, "Parameter identification of linear differential systems via Fourier based modulating functions," *Control-Theory and Advanced Technology*, vol. 1, no. 4, pp. 239-266, December 1985.
- [3] Fiagbedzi, Y. A. and A. E. Pearson, "Feeback Stabilization of linear autonomous time lag systems," *IEEE Trans. on Auto. Contr.*, AC-31, no. 9, pp. 847-855, September 1986.

### 4.2. Conference Proceedings

- [4] Pearson, A. E. and F. C. Lee, "Efficient parameter identification for a class of bilinear differential systems," in *Proc. of IFAC Symp. on Identification and Syst. Param. Est.*, pp. 161-165, University of York, York, UK, July 1985.
- [5] Pearson, A. E. "Order determination for a class of bilinear differential systems," in *Proc. of 1985 ASME-WAM*, DSC-vol. 1, pp. 171-174, Miami, FL, November 1985.

- [6] Fiagbedzi, Y.A. and A. E. Pearson, "Feedback stabilization of state delayed systems via a reducing transformation," in *Proc. of IEEE Conf. on Decis. and Contr.*, pp. 128-129, Ft Lauderdale, FL, December 1985.
- [7] Fiagbedzi, Y.A. and A. E. Pearson, "Finite dimensional approach to tracking in linear time lag systems," in *Proc. of 1986 Conf. on Infor. Sciences and Systems*, pp. 129-134, Princeton University, Princeton, NJ, March 1986.
- [8] Fiagbedzi, Y.A. and A. E. Pearson, "A finite dimensional approach to the feedback stabilization of distributed time-lag systems," in *Preprints of Fourth IFAC Symp on Contr. of Distributed Param. Systems*, UCLA, Los Angeles, CA, June 1986.

## 4.3. Papers Submitted for Publication

- [9] Fiagbedzi, Y.A. and A. E. Pearson, "A multistage reduction technique for feedback stabilizing distributed time lag systems," accepted for publication in *Automatica*
- [10] Fiaghedzi, Y.A. and A. E. Pearson, "Tracking controllers for linear systems with distributed delay," submitted to *Automatica*.
- [11] Pearson, A.E. and Y.A Fiagbedzi, "An observer for time-lag systems," submitted to IEEE for presentation at the 1987 ACC.

\_ / \_\_\_